WEEKLY TEST RANKER'S BATCH TEST - 16 RAJPUR SOLUTION Date 12-01-2020

[PHYSICS]

1. Mutual inductance between two coils

$$M = -\frac{e_2}{(di_1/dt)} = -\frac{e_1}{(di_2/dt)}$$

Also, $e_1 = -L_1 \frac{di_1}{dt}$

$e_2 = -L_2 \frac{di_2}{dt}$

$\therefore$ $M^2 = \frac{e_1 e_2}{\left(\frac{di_1}{dt}\right)\left(\frac{di_2}{dt}\right)} = L_1 L_2$

$\Rightarrow$ $M = \sqrt{L_1 L_2}$

2. Equivalent inductance

$$\frac{1}{L_{eq}} = \frac{1}{L} + \frac{1}{L}$$

$\therefore$ $L_{eq} = \frac{L}{2}$

3. Self-inductance of solenoid is given by

$$\frac{\mu_0 N^2 A}{L}$$

4. Self-inductance of coil is directly proportional to square of number of turns in the coil, *i.e.*,

$$L \propto N^2$$

$\therefore$ $\frac{L_1}{L_2} = \frac{N_1^2}{N_2^2}$

$\therefore$ $\frac{L}{L_2} = \frac{N^2}{(4N)^2}$

$\therefore$ $L_2 = 16L$

5. In the given circuit, three inductances are in parallel, their equivalent inductance,

$$\frac{1}{L_{eq}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \text{ or } L_{eq} = \frac{3}{3} = 1\text{H}$$

6. Mutual inductance between two coils depends on their degree of flux linkage, *i.e.*, the fraction of flux linked with one coil which is when some current passes through the other coil. In figure (A) two coils with their planes are parallel. In this situation, maximum flux passes.

7. $L = \left|\frac{e}{di/dt}\right| = \frac{8}{(2/0.5)} = 2 \text{ H}$

8. $L_P = \frac{L_1 L_2}{L_1 + L_2}$ and $L_S = L_1 + L_2$

9. $M = \frac{\Delta\phi_2}{\Delta i_1}$ because $\phi_2 = Mi_1$

$\therefore$ $\Delta\phi_2 = M(\Delta i_1)$

10. $e_2 = M\left(\frac{di_1}{dt}\right) = i_2 R_2$

$\therefore$ $\left(\frac{di_1}{dt}\right) = \frac{i_2 R_2}{M} = \frac{(0.4)\,(5)}{0.5} = 4 \text{ A/s}$

12. Growth of current in the circuit is given by

$$i = i_0(1 - e^{-Rt/L})$$

where, i_0 is peak value of current and

$$i_0 = \frac{5}{5} = 1 \text{ A}$$

$\therefore$ $i = 1\,(1 - e^{-5 \times 2/10})$

$$= (1 - e^{-1}) \text{ A}$$

13. In case of growth of current in a L-R circuit, the current in the circuit grows exponentially with time 0 to the maximum value $i_0 = E / R$.

14. In one time constant current reduces to $\frac{1}{e}$ times.

15. Current in the circuit $i = V / R = 10/2 = 5\text{A}$

$\therefore$ Magnetic energy stored in the coil

$$U = \frac{1}{2} Li^2$$

$$= \frac{1}{2} \times 2 \times (5)^2$$

$$= 25 \text{ J}$$

16. During growth of current in the coil

$$i = i_0(1 - e^{-Rt/L})$$

For $i = \frac{i_0}{2}$

$$\frac{i_0}{2} = i_0\,(1 - e^{-Rt/L})$$

$\therefore$ $t = 0.693\,\frac{L}{R}$

$$= 0.693 \times \frac{300 \times 10^{-3}}{2} = 0.1 \text{ s}$$

17. Growth of current in the circuit is

$$i = i_0(1 - e^{-Rt/L})$$

$\Rightarrow$ $\frac{di}{dt} = \frac{d}{dt} i_0 - \frac{d}{dt} i_0 e^{-Rt/L}$

$\therefore$ $\frac{di}{dt} = 0 - i_0\left(-\frac{R}{L}\right) e^{-Rt/L} = \frac{i_0 R}{L} e^{-Rt/L}$

Initially, $t = 0$,

$\therefore$ $\frac{di}{dt} = \frac{i_0 R}{L} = \frac{E}{L} = \frac{5}{2} = 2.5 \text{ A/s}$

18. $E = \frac{1}{2} Li^2$

19.

20.

21. $P = F \cdot v = F\ m \cdot v$

$= i\ l\ B \cdot v$

$= \left(\frac{B\ v\ l}{R}\right) \cdot lB \cdot v$

$= \frac{B^2 l^2 v^2}{R}$

22. $V_a - i\ R - L\frac{di}{dt} = V_b$

$\therefore \quad V_a - V_b = i\ R + L\frac{di}{dt}$

According to given conditions :

$8 = 2R + L$ ...(i)

$4 = 2R - L$

Solving these two equations we get,

$R = 3\ \Omega$ and $L = 2\,\text{H}$

23. $V = L\frac{di}{dt}$ or $V \propto L$ $\left(\text{as } \frac{di}{dt} \to \text{same}\right)$

$\therefore \quad \frac{V_1}{V_2} = \frac{L_1}{L_2} = 4$

$P = Vi = \text{constant}$

$\therefore \quad i \propto \frac{1}{V}$

or $\quad \frac{i_1}{i_2} = \frac{V_2}{V_1} = \frac{1}{4}$

$W = \frac{1}{2} Li^2$

$\therefore \quad \frac{W_1}{W_2} = \frac{L_1}{L_2}\left(\frac{i_1}{i_2}\right)^2$

$= (4)\left(\frac{1}{16}\right) = \frac{1}{4}$

24. $\tau_L = \frac{L}{R} = \frac{1}{2}$ second

$i = i_0\,(1 - e^{-t/\tau_L})$

$\therefore \quad \frac{i_0}{i} = \frac{1}{1 - e^{-t/\tau_L}}$

Substituting, $t = 1$ second and $\tau_L = \frac{1}{2}$ s

We get, $\quad \frac{i_0}{i} = \frac{e^2}{e^2 - 1}$

25. For a capacitor,

$i = \left(\frac{dq}{dt}\right) = C \cdot \left(\frac{dV}{dt}\right)$, i.e, i = constant if $\frac{dV}{dt}$ = constant

$\therefore \quad C = \frac{i}{\left(\frac{dV}{dt}\right)} = \frac{1}{2} = 0.5\ \text{F}$

26. $$V = 2t$$

∴ $$L \cdot \frac{di}{dt} = 2t$$

or $$2 \cdot \frac{di}{dt} = 2t$$

or $$\frac{di}{dt} = t$$

or $$(di) = t\,(dt)$$

Integrating, we get

$$i = \frac{t^2}{2}$$

i.e, i-t graph is a parabola.

At $t = 2$ s, $i = 2$ A

∴ $$U = \frac{1}{2} Li^2 = \frac{1}{2} \times 2 \times 4 = 4 \text{ J}$$

$$\frac{dU}{dt} = Li\left(\frac{di}{dt}\right)$$

$$= (2)\left(\frac{t^2}{2}\right)(t) = t^3$$

At $t = 1$ second, $\frac{dU}{dt} = 1$ J/s

27. $$E\,l = \frac{d\phi}{dt}$$

or $$E\,(2\pi R) = \pi R^2 \cdot \frac{dB}{dt} = \pi R^2\,(8t)$$

∴ $$E = 4Rt$$

$$F = qE = 4qRt \qquad \text{(tangential)}$$

$$\tau_F = 4qR^2 t$$

$$\tau_f = (\mu mgR)$$

When $\tau_F > \tau_f$, ring will start rotating.

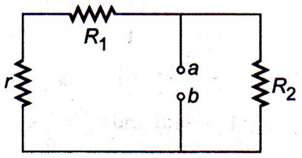
At 2 sec,

$$8qR^3 = \mu mgR$$

∴ $$\mu = \frac{8qR}{mg}$$

28. Since you are decreasing current in the circuit by increasing resistance of the circuit.
Induced emf across inductor will support 20 V battery. Hence, net emf of the circuit is greater than 20 V. Or current in the circuit is more than 4 A.

29. After short circuiting the battery.



$$R_{ab} = \frac{R_2\,(R_1 + r)}{R_2 + R_1 + r}$$

∴ $$\tau_L = \frac{L}{R_{net}}$$

$$= \frac{L\,(R_1 + R_2 + r)}{(R_1 + r)\,R_2}$$

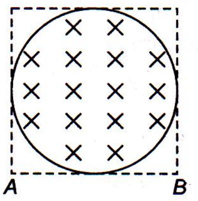
30. $I = \frac{\text{PD}}{\text{Resistance}}$

$\therefore \quad \text{PD} = 2 \times 12 = 24 \text{ V} = B\,v\,l$

Here $l = AD \sin 37° = 0.3 \times \frac{3}{5} = 0.18$ m

$\therefore \quad v = \frac{24}{Bl} = \frac{24}{4 \times 1.08} = \frac{100}{3}$ m/s

31. $V_{ba} = \int \mathbf{E} \cdot d\mathbf{l} = \frac{1}{4}\left(\frac{d\phi}{dt}\right)$

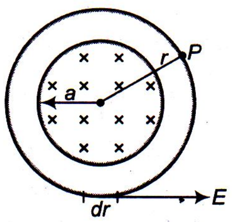


$= \frac{1}{4}(\pi R^2) \cdot \frac{dB}{dt} = \frac{\pi R^2 \alpha}{4}$

32. Magnetic lines on circular loop, due top current in wire AB are tangnetial to its plane.

$\therefore \quad \phi = 0$ or $\frac{d\phi}{dt} = 0$ or induced emf $= 0$.

33. .



Draw a concentric circle of radius r. The induced electric field (E) at any point on the circle is equal to that at P.
For this circle, induced emf

$$e = \oint \mathbf{E}\ d\mathbf{l} = \left|\frac{d\phi}{dt}\right| = A\left|\frac{dB}{dt}\right|$$

$$\therefore \quad E = \oint dl = \pi a^2 \left|\frac{dB}{dt}\right|$$

(but, $\oint dl = 2\pi r$)

$$\therefore \quad E \times (2\pi r) = \pi a^2 \left|\frac{dB}{dt}\right|$$

$$\therefore \quad E = \frac{a^2}{2r}\left|\frac{dB}{dt}\right|$$

$$\Rightarrow \quad E \propto \frac{1}{r}$$

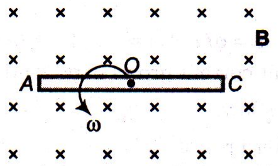
34. According to Fleming's right hand rule P is at higher potential and Q is at lower potential. Therefore, A is positively charged and B is negatively charged.
Also, charge

$$Q = CV = C(Bvl)$$
$$= 10 \times 10^{-6} \times 4 \times 2 \times 1 = 80\,\mu C$$

$\therefore$ $q_A = 80\,\mu C$

and $q_B = -80\,\mu C$

35. For rotating rod, induced emf



$$e = \frac{1}{2} Bl^2\omega$$

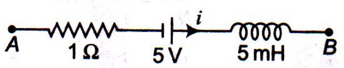
For part AO, $e_{OA} = e_O - e_A = \frac{1}{2} Bl^2\omega$

For part OC,

$$e_{OC} = e_O - e_C = \frac{1}{2} B(3l)^2\omega$$

$\therefore$ $e_A - e_C = 4Bl^2\omega$

36. Applying Kirchhoff's second law to above circuit,



$$V_A - iR + 15 - L\frac{di}{dt} = V_B$$

or $V_A - 5 \times 1 + 15 - (5 \times 10^{-3})(-10^3) = V_B$

$\therefore$ $V_B - V_A = 15$ volt

37. (c) $i = \frac{|e|}{R} = \frac{N}{R}.\frac{\Delta B}{\Delta t} A\cos\theta$

$$= \frac{20}{100} \times 1000 \times (25 \times 10^{-4})\cos 0°$$

$\Rightarrow i = 0.5$ A

38.

$$2\pi v = 377 \Rightarrow v = 60.03\ Hz$$

39.

$$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{120}{1.414} = 84.8\text{ V}$$

40.

Phase difference $\Delta\varphi = \varphi_2 - \varphi_1 = \frac{\pi}{6} - \left(\frac{-\pi}{6}\right) = \frac{\pi}{3}$

41.

The current takes $\frac{T}{4}$ sec to reach the peak value.

In the given question $\frac{2\pi}{T} = 200\pi \Rightarrow T = \frac{1}{100}$ sec

$\therefore$ Time to reach the peak value $= \frac{1}{400}$ sec

42.

$$i_{\text{r.m.s}} = \frac{6}{\sqrt{2}} = 3\sqrt{2}\ A$$

43.

$$i_{\text{r.m.s.}} = \frac{V_{r.m.s.}}{R} = \frac{200}{40} = 5\ A \Rightarrow i_0 = i_{\text{r.m.s.}}\sqrt{2} = 7.07\ \text{A}$$

44.

$$E = E_0 \cos\omega t = E_0 \cos\frac{2\pi t}{T}$$

$$= 10\cos\frac{2\pi \times 50 \times 1}{600} = 10\cos\frac{\pi}{6} = 5\sqrt{3}\ \text{volt.}$$

45.

$$I = I_1 \cos\omega t + I_2 \sin\omega t$$

$$(I^2) = \text{mean} = I_1^2\overline{\cos^2\omega t} + I_2^2\overline{\sin^2\omega t}$$

$$+ 2I_1I_2\overline{\cos\omega t \,.\sin\omega t}$$

$$= I_1^2 . \frac{1}{2} + I_2^2 . \frac{1}{2} + 2I_1I_2 \times 0$$

$$I_{\text{r.m.s.}} = \frac{(I_1^2 + I_2^2)^{1/2}}{\sqrt{2}}$$

[CHEMISTRY]

46. The order of acid strength is RCOOH > HOH > ROH > HC≡CH
Electron-releasing alkyl group R in ROH makes it lesser acidic than H_2O.

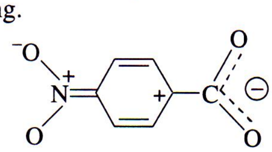
47. Nearer the chlorine to the —COOH group, stronger the acidity. Hence, the correct order is
$CH_3CH_2CHClCOOH > CH_3CHClCH_2COOH > CH_2ClCH_2CH_2COOH$

48. Larger the electronegativity of halogen, stronger the acidity. Hence, the correct order is
$FCH_2COOH > BrCH_2COOH > ICH_2COOH$

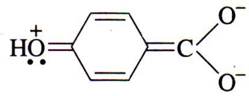
49. Tollens reagent gives white precipitate with methanoic acid and not with ethanoic acid.

50. Electron-releasing group makes benzoic acid a weaker acid while electron-attracting group makes it a stronger acid. Stronger the acid, lesser the value of pK_a°. Hence, the correct order is
$pK_a^\circ(p\text{-}O_2NC_6H_4COOH) < pK_a^\circ(C_6H_5COOH) < pK_a^\circ(p\text{-}HOC_6H_4COOH)$

51. The nitro group interacts with the phenyl ring and thereby induces some positive charge on the ring bearing the COO^- causing a strong electron-withdrawing inductive effects on COO^- group. This effect is base-stabilizing and thus acid strengthening.



52. At *para* position, —OH places negative charge on the carboxylate group and thus making it



weaker than bezoic acid. This effect predominates over its electron-withdrawing acid strengthening inductive effect. However, at *meta* position, there is no such resonance effect and only inductive effect operates and because of its nearness to the COOH group, *meta* isomer is stronger acid than its *para* isomer.

53. *Meta* hydroxy is stronger than *para* isomer (see Q.52). Because of the *ortho effect*, *ortho* isomer is the strongest acid.

54. Increasing crowdiness near the site of esterification decreases the rate of esterification. Hence, the correct order is $1° > 2° > 3°$.

55. Same as Q 54 The correct order is $RCH_2COOH > R_2CHCOOH > R_3CCOOH$

56. α-Hydrogen is replaced by chlorine. The product is $CH_3CH(Cl)COOH$.

57. Heating calcium formate along with calcium benzoate produces benzaldehyde.

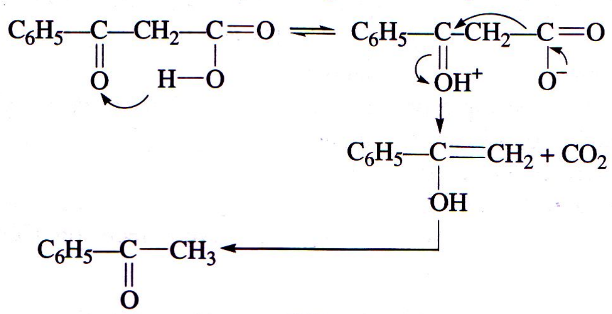
58. The products are CO and H_2O. $HCOOH \xrightarrow{\text{Conc. } H_2SO_4} CO + H_2O$.

59. The Hell-Volhard-Zelinsky reaction is used in the synthesis of α-haloacids.

60. In p-$CH_3COC_6H_4COOH$, the CO group is also reduced by $LiAlH_4$.

61. Succinic acid ($HOOCCH_2CH_2COOH$) gives cyclic anhydride on heating.

62. *β*-Ketoacids are unstable acids. These readily undergo decarboxylation through a cyclic transition state.



63.

64. Bromine is lesser electronegative than fluorine. Bromine attached to β-carbon will cause least enhancement in the dissociation of halo substituted propanoic acid.

65. Carboxylic acids have relatively high boiling points because of hydrogen bonds. Their boiling points are somewhat higher than those of alcohols of comparable molar masses. Since there exists no hydrogen bondings in aldehydes, their boiling points are lower than those of corresponding alcohols.

66. BH_3/THF followed by H_3O^+ reduces only —COOH group without affecting —CO— group. $LiAlH_4$ reduces only —COOH group without affecting C=C group.

67. The order of reactivity is acid chloride > anhydride > ester.

68. The correct order is $Cl^- < RCOO^- < RO^-$.

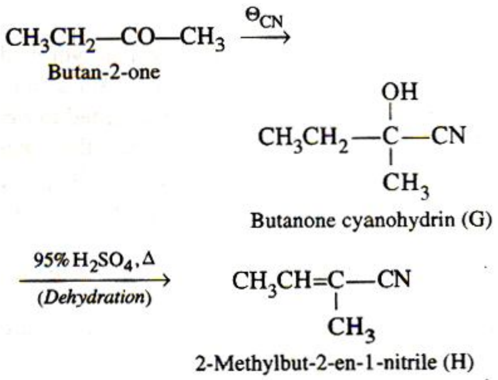
69. The correct order is $Cl^- < RO^- < NH_2^-$.

70. Electron-withdrawal group increasing reactivity of hydrolysis while the electron-releasing group decreases reactivity. The correct order is p-$O_2NC_6H_4COCl > PhCOCl > p$-$CH_3OC_6H_4COCl$.

71. (a)
72. (a)
73. (b)
74. (b)

75. (a)

76.



Please note that hydrolysis of cyanides to carboxylic acids requires addition of a molecule of H_2O. *Since 95% H_2SO_4 cannot supply H_2O, therefore, dehydration of (G) occurs to give (H).*

77.

Due to ortho-effect, *o*-nitrobenzoic acid is the strongest acid. Further since electron-withdrawing groups such as NO_2 increase while electron-donating groups such as OH decrease the acid strength *w.r.t.* parent acid, therefore, the overall acidity increases in the order : C < D < B < A, *i.e.*, option (*d*) is correct.